Physical Observation of a Robust Acoustic Pumping in Waveguides with Dynamic Boundary

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Research on breaking time-reversal symmetry to realize one-way wave propagation is a growing area in photonic and phononic crystals and metamaterials. In this Letter, we present physical realization of an acoustic waveguide with spatiotemporally modulated boundary conditions to realize nonreciprocal transport and acoustic topological pumping. The modulated waveguide inspired by a water wheel consists of a helical tube rotating around a slotted tube at a controllable speed. The rotation of the helical tube creates moving boundary conditions for the exposed waveguide sections at a constant speed. We experimentally demonstrate acoustic nonreciprocity and topologically robust bulk-edge correspondences for this system, which is in good agreement with analytical and numerical predictions. The nonreciprocal waveguide is a one-dimensional analog to the two-dimensional quantum Hall effect for acoustic circulators and is characterized by a robust integer-valued Chern number. These findings provide insight into practical implications of topological modes in acoustics and the implementation of higher-dimensional topological acoustics where time serves as a synthetic dimension.

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The law of reciprocity is a fundamental principle and design constraint of waves and vibrations control in linear, time-invariant systems that require energy transport between two points in space to be symmetric. Thus, reciprocity can be a hindrance in systems where asymmetric wave motion is sought. For example, one-way acoustic devices, such as three-port circulators [1] and one-way diodes [2], cannot exist with unbroken reciprocity [3–8]. Creating nonreciprocal materials and devices has therefore attracted tremendous attention for enabling unidirectional energy transport [9–17], topological insulators [18,19], and edge modes [20,21]. Many strategies, including strongly nonlinear networks [22–26], gyroscopic media [27,28], circulating fluids [29,30], and spatiotemporally modulated materials [31–39], have been proposed. Among them, spatiotemporal modulation by introducing a temporal topological pumping has been widely investigated in active elastodynamic systems. However, the physical realization of the temporal pumping generally requires sophisticated external fields or smart materials applied in the system. For example, nonreciprocity induced by electromechanical pumping was recently experimentally demonstrated by introducing a periodic array of electromagnets on a beam to control positive and negative propagating mechanical waves [31]. A similar approach was adopted experimentally by proposing a magnetomechanical topological insulator to demonstrate the temporal pumping that produces robust mechanical energy transport [38]. Surprisingly, compared with their mechanical counterparts, there are few works on physical evidence of spatiotemporal pumping in acoustics, not to mention the nonreciprocal and topological transport of acoustic waves.

In this Letter, we propose a 1D dynamic waveguide that displays nonreciprocal wave phenomena without imposing external fields to modulate the acoustic properties in space and time. Rather, we employ a novel paradigm whereby the bulk properties are left intact, while the boundary conditions are, through purely mechanical means, modulated in space and time. This is achieved by mounting a helical tube on a slotted acoustic waveguide and then rotating the helix with a motor, as shown in Fig. 1(a). This configuration is shown to be capable of pumping acoustic energy unidirectionally in a manner reminiscent of a water wheel. The system results in an acoustic waveguide with exposed holes (shunts) that are effectively translated at a constant speed along the axis of the waveguide when the helical tube is rotated by the motor. It is worth noting that, while the properties of the tube are time independent, the boundary conditions and therefore the impedance of the waveguide is indeed time dependent, leading to the “dynamic” waveguide [31]. This strategy differs from the acoustic pumping where wavelike modulations of constitutive parameters are utilized to study parametric amplification in the unstable regime [40]. As a result, the modulated acoustic waveguide
FIG. 1. (a) Schematic illustration of the modulated acoustic waveguide. A fixed, slotted tube (gray) with a long straight opening is inserted into a helical tube (blue), forming an array of parallelogram openings in the acoustic waveguide. The helical tube is driven mechanically by a motor system. Clockwise and counterclockwise rotations realize forward and backward spatiotemporal modulation, respectively, as the exposed sections translate, indicated by the insets. (b) Schematic of the equivalent spatiotemporal modulation, respectively, as the exposed sections translate, indicated by the insets. Rotation of simulations.

We utilize the transfer matrix method to visit the acoustic nonreciprocal regime for various modulation speeds $c_m$ (see Supplemental Material [42], Sec. A for derivation details). The effect of slow modulations (the modulation frequency is smaller than the bandwidth) is to shear the dispersion curves and create a couple of directional band gaps. The emergence of the direction gaps is the direct observation of the band-tilting-induced nonreciprocity.

Progressive modulation in various modulation frequency regions creates a bias in space and time, which enables nonreciprocal wave propagation as a function of modulation frequency and depth. For a small modulation speed, the phase shift induced by adiabatic pumping over a short time period is insufficient to trigger any intermodal transitions and the band structure gets sheared and tilted to create a directional band gap [8,43]. For a small-amplitude and moderate modulation speed, Bragg scattering due to time-dependent interface causes the frequency shift for scattered waves, which is related to Doppler shifting [45]. Figure 2(a) illustrates the theoretical dispersion relations of the modulated waveguide with a small modulation speed $c_m = 0.029c_0$, where the modulation frequency ($\omega_m = 20 \text{ rad/s}$) is much smaller than the width of the first band gap ($\delta\omega = 160 \text{ rad/s}$). In this example, the geometric parameters are $L_1 = 0.01 \text{ m}$, $L_2 = 0.03 \text{ m}$, $S = L_2^2 = 1 \times 10^{-4} \text{ m}^2$, $\sigma_1 = 0.25 \times 10^{-2} \text{ m}$, and $A = 1.54 \times 10^{-4} \text{ m}^2$. The air density, bulk modulus, and mass of air in the exposed sections are $\rho = 1.21 \text{ kg m}^{-3}$, $B = 1.42 \times 10^5 \text{ Pa}$, and $M=1.21 \times 10^{-7} \text{ kg}$, respectively. In Fig. 2(a), we notice that the lower boundary at $kL = 0$ does not shift, indicating that the first cutoff frequency is independent of the modulation speed. This is because the first cutoff frequency...
corresponds to a stationary wave at $k = 0$ and therefore is insensitive to the modulation when the modulation speed is slow. The effect of the modulation speed on the cutoff frequency has been investigated in detail [43]. However, its upper boundary exhibits a frequency shift of $c_m/L$. Accordingly, the band gaps become directional, resulting in nonreciprocal wave propagation at these frequencies. Frequency shifts of the second and third passbands are also observed. The continuous modulation-induced tilting of the dispersion curves can be interpreted by the adiabatic theorem for slow modulation. The frequency shift is determined by $C \omega_m$, where $C$ is the Chern number (Supplemental Material [42], Sec. B). To validate the theoretical prediction, numerical simulations for the modulated system (Fig. 1) are conducted using a finite-element method (FEM)-based software COMSOL Multiphysics.

In the simulations, a five-cycle tone burst signal $p = [1 - \cos(0.5 \pi \tau)] \sin(2 \pi \tau) / 5 \text{kHz}$ is applied at one end of the tube and the time-domain transmitted signal is measured at the other end. Based on Fourier transform, Fig. 2(b) illustrates the frequency spectra of the incidence and transmission with $c_m = 10 \text{ m/s}$ and without modulation. To reduce the undesired reflection, time-domain perfect matching layers are applied on both ends of the tube (Supplemental Material [42], Sec. C). As shown in Fig. 2, the acoustic nonreciprocity is observed around the lower and upper boundaries of the first stop band, shifting the first branch up to $c_m/L$ compared with the nonmodulated case. A possible realistic experimental setup and the associated measuring procedure are discussed in Sec. C of the Supplemental Material [42]. In addition, the acoustic wave propagation at other frequencies are tilted with transmitted amplitudes unaltered. The time-domain numerical results demonstrate that the acoustic pumping is sufficient to break reciprocity when traveling along the modulation direction and suppressed in the other direction.

We experimentally demonstrate the acoustic band-tilting-induced nonreciprocity using a purely mechanical test bed, as shown in Fig. 3. The test bed, fixed to a vibration isolation platform by 3D printed supports, is a full realization of the numerical model illustrated in Fig. 1. Both the slotted and helical tubes are made from aluminum. The inner slotted tube and outer helical tube have inner radii of 7 and 8.2 mm and outer radii of 8 and 9.3 mm, respectively. The helical tube is rotated by a motor. We used Mecanum’s four microphone impedance tubes and collected the data with a B&K 3050-A data acquisition (DAQ) system. While the helical tube rotates, the ESs translate along the tube axis (see video in the Supplemental Material [42]). Acoustic signals are generated with a loudspeaker on the right side of the tube. Four acoustic transducers, working as probes, are placed to collect the reflected and transmitted signals (see Fig. 3). The contact areas of the two tubes are filled with lubricant to prevent sound leakage and reduce noise.

We first measured the sound transmission by translating the ESs characterized by moving boundary conditions. The loudspeaker is driven by a swept-sine input covering 1–5 kHz. When the rotation is sufficiently slow to satisfy the adiabatic condition, the bulk bands of the modulated tube are tilted with respect to their static reference configuration (see also Fig. 2). A directional band gap appears around 4450 Hz with a small modulation speed of $c_m = +10 \text{ m/s}$. This enables the sound to transmit within the frequency range of $4450 - (4450 + c_m / 2L) \text{ Hz}$, which is forbidden when the tube is static. In other words, nonreciprocal sound transport is triggered within the band gap when the tube is rotating; see Fig. 4(a). As expected, good agreement between numerical and experimental results is observed. The nonreciprocal frequency region is marked by the blue-shaded area in Fig. 4(a), where the clockwise (CW) and counterclockwise (CCW) modulations display asymmetric transmission with a bandwidth of

![FIG. 2. (a) Analytically calculated dispersion diagrams of the modulated effective medium with $c_m = 10$ (solid blue), 0 (solid black), and $-10 \text{ m/s}$ (solid red). A frequency shift of the band gap lower bound $\Delta f = c_m / L$ is obtained when modulated from $c_m = 10$ to $c_m = -10 \text{ m/s}$. (b) Normalized sound amplitudes from numerical simulations for modulation speeds $c_m = 10$ (solid blue) and $c_m = -10 \text{ m/s}$ (solid red). The solid black curve represents the input signal.](image)

![FIG. 3. Experimental setup of the spatiotemporal modulated acoustic system. The test bed is composed of three main components: a tube system composed of the slotted tube and exposed helical tube, a gear system and motor to drive the rotation of the outer helical tube, and an acoustic testing system containing a loudspeaker and four acoustic transducers (probes).](image)
Δf = c_m/L. To better quantify the nonreciprocal behavior, numerically and experimentally attained transmission spectra under harmonic loading at \(f_e = 4450\) Hz is plotted in Fig. 4(b). The magnitudes of sound transmission at the excitation frequencies (within the modulation passband) are significantly greater than those for the negative modulation (within the modulation band gap). Moreover, the intensities of lower harmonics (in the passband) are much stronger than those of the higher ones (within the band gap). Further, high-order harmonics are observed as additional peaks, with multiple frequencies \(f = f_e \pm n\Delta f\), where \(\Delta f = 250\) Hz, and \(n = 0, 1, 2, \ldots\). These extra harmonics are the consequence of the traveling waves being scattered by the moving boundaries as in the Doppler effect [6]. It should be mentioned that, in spite of the lossy characteristics of the ESs, the nonreciprocal behavior is still noticeable since it is quantized by a robust Chern number (Supplemental Material [42], Secs. B and C).

To observe the topological pumping, we conduct experiments by turning the tube under different boundary conditions to examine the bulk-edge correspondence principle through a pumping cycle (Supplemental Material [42], Sec. D). The test bed is identical to what has been shown in Fig. 3, except the helical tube is manually turned to certain position instants and remains static. A microphone with subwavelength dimension is placed inside the tube for scanning the pressure field distribution along the tube axis (Supplemental Material [42], Sec. D). Harmonic excitations of different frequencies are then applied when the phase of the modulated tube is swept over one complete rotation of \(2\pi\) by manually translating the ESs from 0 to 20 mm (Supplemental Material [42], Sec. D). The measured pressure field distributions at three different position instants, corresponding to the three highlighted eigenmodes in Fig. 5(a), are presented in Fig. 5(b) by symbol plots. For the bulk mode around 4.4 kHz illustrated in Fig. 5(b)(i), except for the magnitude decrease due to sound attenuation, a standing wave profile is still observable describing the bulk mode. The right (under the left excitation) and left (under the right excitation) edge modes are shown in Fig. 5(b)(ii) and (iii), respectively. The establishment of the acoustic localization under the excitations on the opposite sides proves the existence of the
topological edge modes (Supplemental Material [42], Sec. D). Eigenfrequency analysis also provides the acoustic pressure distributions at these position instants (i)–(iii), as shown in Fig. 5(c). Good agreement with the measured normalized acoustic distributions is evidenced. Finally, to directly validate the localized edge mode, broadband acoustic harmonic tests are implemented by applying acoustic excitation on both the right and left sides of the system under phase condition (ii) (see Supplemental Material [42], Sec. D for the measured frequency spectrum of the edge mode). The results confirm the emergence of the topologically protected edge mode localized around the right end of the tube under the current phase condition.

In summary, we have experimentally demonstrated the mechanically modulation-induced nonreciprocity in acoustics and the adiabatic topological pumping phenomena, associated with the purely mechanical realization of spatiotemporally modulated acoustic waveguide. The spatiotemporal modulation behaving as an acoustic pumping strategy offers unprecedented control and reconfigurability over the acoustic energy transport in space and even in frequency dimensions. We believe that the proposed mechanical realization of the acoustic modulated nonreciprocal systems paves the way for realizing topological phononic logic and acoustic energy localization and trapping applications.

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