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A novel non-linear cumulative fatigue damage model based on the degradation of material memory

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Abstract

Many fatigue damage models have been investigated based on the *S–N* curve or modified *S–N* curve; however, a number of them require additional efforts to determine the material parameters or do not consider the loading history (loading interactions, loading sequences, loading levels, etc.). These limitations can result in extreme deviations for estimating the fatigue life in real-world scenarios. To address these limitations, a new fatigue damage model is developed based on the material memory, which can be described as the degradation of mechanical properties under cyclic loadings. Comparisons with three models are used to demonstrate the validity of the proposed model. Furthermore, four sets of experimental data under two-stress and four-stress levels are carried out to verify the validation of the proposed model, which improves the residual life estimation over the three existing models used for comparison.

Keywords

Fatigue life, damage accumulation, residual life estimation, material memory, loading history

Introduction

Fatigue failure is a very common concern in practical engineering applications, which has been studied extensively since the 19th century. The estimations of fatigue damage and fatigue life play

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important roles in fatigue failure analysis. Because of the variable working conditions, it is very difficult to describe the damage accumulation accurately in most real-world cases (Aeran et al., 2017; Fatemi and Yang, 1998; Huang et al., 2017; Li et al., 2018a, 2018b, 2019; Liu et al., 2018; Lv et al., 2015; Mi et al., 2018; Peng et al., 2016; Rao et al., 2001; Zhang et al., 2018). Fatigue damage increases with continued loadings, and will result in the degradation of mechanical characteristics and subsequent failures. Fatigue damage is affected by the loading histories, where the damage accumulates and strength degrades, and the residual life under cyclic loadings becomes limited. To predict the life of engineered structures, an assessment or a model for damage accumulation is required. In the past two centuries, many models and assessments have been proposed to describe the damage accumulation process, which can be roughly divided into two approaches: (1) linear damage rules (LDR) and (2) non-linear damage rules (NLDR).

Miner's rule is the most popular cumulative theory applied to the engineering practice of fatigue approximation/modeling. However, it is not accurate and has many shortcomings in addressing the damage accumulation process. Many models and theories have been established or modified based on Miner's rule to improve the prediction accuracy (Fatemi and Yang, 1998). NLDR is more applicable than LDR under various loading conditions and has become a popular research area in practice (Huang et al., 2019; Liu et al., 2019; Sun et al., 2017; Wang and Liu, 2019). Aeran et al. (2017) proposed a non-linear damage model without considering any additional material parameters and considered the loading interactions by introducing an interaction factor based only on the related stress. Lv et al. (2015) and Peng et al. (2016) have also introduced the interaction factor to account for the loading interactions in the fatigue damage models. EI-Aghoury and Galal (2013) introduced virtual life target curves to describe the damage accumulation process and provided a visually illustrative method to estimate the life cycles under various loading conditions. Benkabouche et al. (2015) provided a useful damage model, which extended the uniaxial loading to multiaxial loading based on finite element analysis. Zuo et al. (2015) described the fatigue accumulation process by introducing the driving stress based on the *S–N* curve, which can address problems with loading history. Mao and Mahadevan (2002) established a mathematical tool to assess the damage evolution process, which can quantify the rapid degradation of material properties at the early and final life cycles. Jinescu (2013) developed a critical energy approach to calculate the fatigue damage, which considered the non-linear characteristic of materials, mean stress, loading rates, and residual stress. Chen et al. (2006) proposed a modified approach to consider the non-proportional loading effects under sequential biaxial loadings, and the predictions have shown good agreements with all the experimental data.

Fatigue damage accumulation models

Linear damage rule

Because of the advantages of LDR (Miner's rule), which has a simple form and a few parameters, it is the most widely employed fatigue approximation technique used in industrial and engineering applications. The mathematical form of Miner's rule is given as (Fatemi and Yang, 1998)

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} \tag{1}$$

where D denotes the total damage, n_i and N_i are the number of loading cycles and life cycles at the ith load stress level σ_i, respectively.

The main shortcoming of LDR is that the loading history is not considered when calculating the damage. The rate of damage accumulation is irrelevant to the loading history, and fatigue damage is accumulated only when the stress is equal to or greater than its fatigue limit. This means that it is non-conservative in various working conditions, especially when the stress is below the fatigue limit and it will have no contribution to the damage accumulation based on Miner's rule, but damage continues to increase actually. In fact, the loading history has significant effects on the life prediction. The low-amplitude stress lower than its fatigue limit is one of the main reasons for damage accumulation, but Miner's rule ignores it. Moreover, low-amplitude stress may have strengthening effects on the material properties.

Non-linear damage rule

The initial crack and crack propagation, non-linear accumulation effects with various loadings, the degradation of the fatigue limit after prior damage, and the effects of mean stress for fatigue limit or *S–N* curve, should be considered in the damage accumulation and fatigue life prediction (Chaboche, 1988). The loading history obviously affects the estimation of life cycles and damage accumulation. For example, a low–high loading sequence often leads to longer life than reverse loadings. The loadings interaction effects may result in the damage increment of the same engineering structure under various loadings, which differ from the damage increment under constant loadings. The rate of degradation for material properties is related to the stress levels and number of loading cycles. It is not always the case that the high–low sequence will lead to larger damage than the low–high sequence (Paepegem and Degrieck, 2002; Schaff and Davidson, 1997), which depends on the working conditions, but the loading history causes changes to the residual life.

(1) Toughness exhaustion model (Ye's Model) and the modified model (Lv's Model)

Ye and Wang (2001) found that static toughness of the mechanical property parameters had a significant change during the damage evolution and reduction of the static toughness and reflected the inherent energy absorbing ability in the material. A toughness exhaustion model, based on exhaustion and dissipation of static toughness and cyclic plastic strain energy during the fatigue evolution, was proposed and given as

$$D = -\frac{1}{\ln N}\ln\left(1 - \frac{n}{N}\right) \tag{2}$$

This model represents the decline of material mechanical behaviors, and has a good performance in life prediction and damage accumulation; however, it does not consider the loading interaction effects if the structure is under variable amplitude cyclic loadings. Lv et al. (2015) modified this model by introducing the loading interaction term ($\sigma_i/\sigma_{\max}$), which can give more accurate predictions and is given as (Lv's model)

$$D = -\frac{1}{\ln N}\ln\left(1 - \frac{n}{N}\right) \cdot \left(\frac{\sigma_i}{\sigma_{\max}}\right) \tag{3}$$

where $\sigma_{\max}$ is the maximum stress.

(2) Kwofie's model

Kwofie and Rahbar (2013) introduced a new concept (driving stress) to estimate the residual life cycles only in accordance with the *S–N* curve. The proposed method is a function of the expended life fraction of the corresponding stress, which has better agreements with test data and is only related to the loading cycles or life cycles. The cumulative fatigue damage can be calculated by

$$D = \sum \frac{n_i}{N_i} \frac{\ln(N_i)}{\ln(N_1)} \text{ or } \sum \frac{n_i}{N_i} \frac{\ln(\sigma_i/A)}{\ln(\sigma_1/A)} \tag{4}$$

where N_1 is the failure life cycles under the initial applied loading level σ_1, and A is the material constant.

(3) Corten–Dolan model

Corten and Dolan (1956) established a model which was similar to the fracture mechanics approaches. It could describe the fatigue damage in microcosm, which treated the damage nuclei as the cause for initiation of damage where the nuclei increased with the increasing stress values. In fact, the Corten–Dolan model was a way to evaluate fatigue damage based on the modified *S–N* curve, which took into account the loading interactions and was given as

$$D = mrn^a \tag{5}$$

where m is the number of damage nuclei, r is the damage propagation rate, and a is the damage propagation exponent.

When the material reaches failure, the damage can be expected as

$$D = \sum_{i=1}^{k} \frac{n_i}{N_{f\max}} \left(\frac{\sigma_i}{\sigma_{\max}}\right)^d = 1 \tag{6}$$

where $N_{f\max}$ denotes the number of cycles to failure at the highest stress, and d is the material constant, which requires additional endeavors to determine.

The term $(\sigma_i/\sigma_{\max})^d$ in the Corten–Dolan model plays a significant role in reflecting the loading interaction effect. In fact, the factors to consider the loading interaction effect always have been proposed involving the applied stress or adjoining stress with some material constants, such as Morrow's model (Morrow, 1986), Rege's model (Rege and Pavlou, 2017), Lv's model (Lv et al., 2015), which will accelerate or decelerate the damage accumulation process (Calderon-Uriszar-Aldaca and Biezma, 2017; Lin et al., 2018).

The proposed model based on the degradation of material memory

In general, fatigue damage is a very complex process, which involves applied stress, geometric shape, material property, loading cycle, frequency, temperatures and so on, is defined as (Hwang and Han, 1986; Kujawski and Ellyin, 1988)

$$D = F(n, \sigma, f, T, \cdots) \tag{7}$$

where f is the frequency and T is temperature. If the geometric shape and working conditions are determined, equation (7) can be simplified as

$$D = F(n, \sigma) \tag{8}$$

Then, the increment of damage can be obtained by

$$dD = F(dn, \sigma) \tag{9}$$

It is assumed that there is a linear function between the relative increment of damage and the relative increment of loading cycles, so

$$\frac{dD}{D} = f(\sigma)\frac{dn}{n} \tag{10}$$

and $f(\sigma) \geq 0$ for any stress σ, because that damage is an irreversible process. $f(\sigma)$ can be determined for a given stress σ, and integration of equation (10) yields

$$\begin{gathered} \int \frac{dD}{D} = \int f(\sigma)\frac{dn}{n} \\ \ln D = f(\sigma)\ln n + C \end{gathered} \tag{11}$$

where C is a constant of integration. When the materials are up to failure, $n = N$ and $D = 1$, then taking into account this boundary condition, we can get

$$C = -f(\sigma)\ln N \tag{12}$$

Combining equations (11) and (12), we obtain

$$D = \left(\frac{n}{N}\right)^{f(\sigma)} \tag{13}$$

For the two-level block loadings, the structure or component is subjected to the stress σ_1 for n_1 cycles, and to the stress σ_2 for n_2 cycles until failure. In accordance with the equivalent damage theory, the damage caused by the first class can be equivalent to the damage caused by σ_2 for n_{12} cycles, as shown in Figure 1, $D_A = D_B$, we obtain residual life cycle ratio n_{2p}/N_2

$$\begin{aligned} \left(\frac{n_1}{N_1}\right)^{f(\sigma_1)} &= \left(\frac{n_{12}}{N_2}\right)^{f(\sigma_2)} \\ \frac{n_{12}}{N_2} &= \left(\frac{n_1}{N_1}\right)^{f(\sigma_1)/f(\sigma_2)} \\ \frac{n_{2p}}{N_2} &= 1 - \frac{n_{12}}{N_2} = 1 - \left(\frac{n_1}{N_1}\right)^{f(\sigma_1)/f(\sigma_2)} \end{aligned} \tag{14}$$

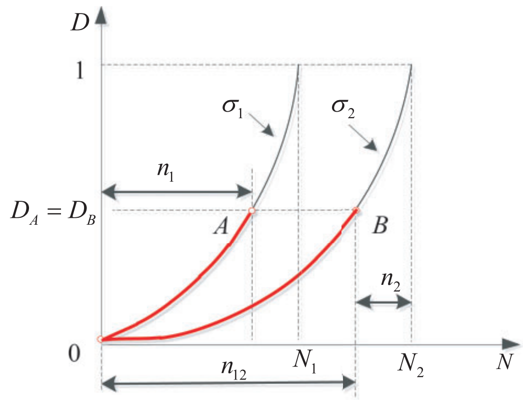

Figure 1. The equivalent damage schematic.

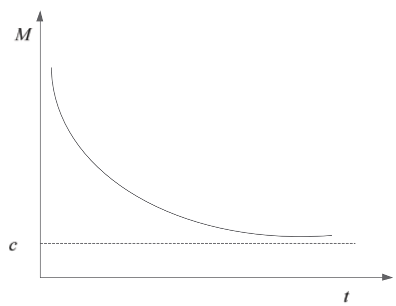

Figure 2. The Ebbinghaus forgetting curve.

For the multi-level block loadings, the residual life cycle ratio n_{i+1p}/N_{i+1} at σ_{i+1} can be given as

$$\begin{gathered}\frac{n_{i+1p}}{N_{i+1}} = 1 - \left\{ \frac{n_{ip}}{N_i} + \left[\frac{n_{i-1p}}{N_{i-1}} + \cdots + \left(\frac{n_{2p}}{N_2} + \left(\frac{n_1}{N_1} \right)^{\alpha_1} \right)^{\alpha_2} \right]^{\alpha_{i-1}} \right\}^{\alpha_i} \\ \alpha_i = \frac{f(\sigma_i)}{f(\sigma_{i+1})}, \quad (i = 1, 2, 3, \cdots)\end{gathered} \tag{15}$$

It is known that the memorized information decreases as time passes. Böhm et al. (2014) took advantage of the material memory and proposed a new accumulative damage model. The Ebbinghaus forgetting curve can be used to characterize the memory, as shown in Figure 2 and equation (16)

$$M(t) = (a - c)e^{-bt} + c \tag{16}$$

where $M(t)$ is the total memorized information; a and b denote the memorization and forgetting factors, respectively; c and t denote asymptote and time, respectively.

Peng et al. (2018) also employed the material memory to model the changes of residual S–N curve and proposed a non-linear cumulative damage rule based on the S–N curve which does not involve any stress considerations. Applying the memory degradation to the fatigue material properties, the material properties degrade with the increasing loading cycles where the material memory function can be written as

$$M(n) = (a - c)e^{-\frac{n}{N}} + c \tag{17}$$

The accumulation process of fatigue damage is corresponded with the change process of material memory, we can get

$$\frac{\Delta M(n)}{\Delta M(N)} = \frac{M(0) - M(n)}{M(0) - M(N)} = \frac{1 - e^{-\frac{n}{N}}}{1 - e^{-1}} \tag{18}$$

where a and c are eliminated. When $n = 0$, $n/N = 0$ and $\Delta M(n)/\Delta M(N) = 0$; when $n = N$, $n/N = 1$ and $\Delta M(n)/\Delta M(N) = 1$. $\Delta M(n)/\Delta M(N)$ has the some boundary and monotonicity with n/N, which increases with the increasing number of loading cycles, and it can be used to model the fatigue damage.

It is known that many cumulative damage models are based on the S–N curve, which is established under constant loadings and thus the curve changes with different stress ratio. Therefore, these models do not consider the mean stress effect if the stress ratio changes with different loading blocks. One of the most popular methods to correct the mean stress effect is the SWT model, which is given as (Smith et al., 1970)

$$\sigma_{eq} = \sqrt{\sigma_{\max}\sigma_a} == \sigma_{\max}\sqrt{\frac{1-R}{2}} = \sigma_a\sqrt{\frac{2}{1-R}} \tag{19}$$

where σ_{eq}, σ_a, and R denote the equivalent stress, stress amplitude, and stress ratio, respectively.

Accounting for the loading interaction and mean stress effect in the damage accumulation, the interaction factor α_i is defined as

$$\begin{cases} f(\sigma_i) = \zeta\left(\sigma_{eq,i}\right)^{\delta} = \zeta\left(\sqrt{\sigma_{\max,i}\sigma_{a,i}}\right)^{\delta} \\ \alpha_i = \frac{f(\sigma_i)}{f(\sigma_{i+1})} = \left(\frac{\sigma_{eq,i}}{\sigma_{eq,i+1}}\right)^{\delta} \end{cases} \quad (i = 1, 2, 3, \cdots) \tag{20}$$

Then, replacing the term n/N in equation (13) by $\Delta M(n)/\Delta M(N)$, the new fatigue damage model is defined as

$$D_n = \left(\frac{\Delta M(n)}{\Delta M(N)}\right)^{f(\sigma)} = \left(\frac{M(0) - M(n)}{M(0) - M(N)}\right)^{f(\sigma)} = \left(\frac{1 - e^{-\frac{n}{N_f}}}{1 - e^{-1}}\right)^{f(\sigma)} \tag{21}$$

In addition, the new model shows a non-linear relation between the total damage and life cycles, where the larger difference between the adjacent loads is, the more significant the non-linearity is. For the two-level block loadings, the prediction of the residual life cycle ratio n_{2p}/N_2 is given by

$$\frac{n_{2p}}{N_2} = 1 - \frac{n_{12}}{N_2} = \ln\left(1 - \left(\frac{1 - e^{-\frac{n_1}{N_1}}}{1 - e^{-1}}\right)^{\alpha_1}\left(1 - e^{-1}\right)\right) + 1 \tag{22}$$

For three-level block loadings, the prediction of the residual life cycle ratio n_{3p}/N_3 under stress level can be estimated by

$$\frac{n_{3p}}{N_3} = \ln\left(1 - \left(\frac{1 - e^{-\left(\frac{n_2}{N_2}+1-\frac{n_{2p}}{N_2}\right)}}{1 - e^{-1}}\right)^{\alpha_2} \left(1 - e^{-1}\right)\right) + 1 \tag{23}$$

For multi-level block loadings, following the same steps before, the residual life cycle ratio n_{ip}/N_i can be derived as

$$\frac{n_{ip}}{N_i} = \ln\left(1 - \left(\frac{1 - e^{-\left(\frac{n_{i-1}}{N_{i-1}}+1-\frac{n_{i-1p}}{N_{i-1}}\right)}}{1 - e^{-1}}\right)^{\alpha_{i-1}} \left(1 - e^{-1}\right)\right) + 1 \tag{24}$$

Determination of the function α_i

Assuming that the parameters (ς, δ) in equation (20) can be determined under two-level block loading, the experimental data of 30NiCrMoV12 were employed to fit (ς, δ). For 30NiCrMoV12, the two-level block loading fatigue tests, including high–low (485–400, 465–420, and 450–420 MPa) and low–high (400–485, 465–420, and 450–420 MPa) stress levels, were carried out on a servo-hydraulic MTS810 testing system under stress-control mode with $R = -1$ and applied 18 specimens (Dattoma et al., 2006).

Under two-level block loading, assuming that the predicted life n_{2p} is equal to the real life n_2, we can obtain

$$\frac{n_2}{N_2} = \ln\left(1 - \left(\frac{1 - e^{-\frac{n_1}{N_1}}}{1 - e^{-1}}\right)^{\alpha_1} \left(1 - e^{-1}\right)\right) + 1 \tag{25}$$

Then, the interaction factor α_1 can be obtained

$$\alpha_1 = \frac{\ln[(1 - e^{-(1-\frac{n_2}{N_2})})/(1 - e^{-1})]}{\ln[(1 - e^{-\frac{n_1}{N_1}})/(1 - e^{-1})]} \tag{26}$$

Furthermore, based on equation (20), the value of δ can be estimated

$$\delta = \frac{\ln \alpha_1}{\ln\left(\sigma_{eq,1}/\sigma_{eq,2}\right)} \tag{27}$$

For a given ratio $\sigma_{eq,1}/\sigma_{eq,2}$, the δ can be estimated by repeatedly applying equation (27) for the 18 specimens with different loading cycles, then calculating the mean value for 18 specimens, finally we can get that $\delta = -5.78$. In fact, the ς will be eliminated when calculating the residual life under multi-level block loadings and only δ is required.

The loading interaction factor can be found to be

$$\alpha_i = \frac{f(\sigma_i)}{f(\sigma_{i+1})} = \left(\frac{\sigma_{eq,i}}{\sigma_{eq,i+1}}\right)^{-5.78}, \quad (i = 1, 2, 3, \cdots) \tag{28}$$

Furthermore, we extend the special case to a general situation. It is assumed that α_i can satisfy the accumulative damage results for different materials (not only for 30CrMnSiA) and which will be verified in the following section. When employing the proposed model to predict the residual life, the stress and the fatigue life should be known. If the materials or structures serve in the more complex situations, the $\sigma_{eq,i}$ can be replaced by other equivalent factors and the relationship between the equivalent factors and fatigue life should be known in advance, but the δ may be changed according to the actual conditions.

Validation and discussion

To confirm the validity and applicability of the proposed model, comparisons with Miner's rule, Lv's model and Kwofie's model are used to predict the residual life. Three cases are employed for C45 steel, Al 2024-T42 and 30CrMnSiA under two-level block loading, and one case is employed for Al 6082-T6 under multi-level block loading.

(1) C45 steel

The experimental data of C45 steel was obtained under the high–low (331.5–284.4 MPa) and low–high (284.4–331.5 MPa) stress levels with $R = -1$, reported by Shang and Yao (1999). The fitted S–N curve of C45 based on the experimental data is shown in equation (29). The prediction results n_{2p} of the residual life cycles by Miner's model, Lv's model, Kwofie's model, and proposed model compared with experimental data n_{2t} are shown in Table 1 and Figure 3(a), where the subscripts t and p represent the tested life and predicted life, respectively

$$\ln \sigma_{\max} = -0.06655 \times \ln N + 6.524 \tag{29}$$

Table 1. Residual lives of C45 steel under two-level block loading.

Loading sequence	Experimental data			Miner's rule	Lv's model	Kwofie's model	Proposed model
$\sigma_{\max}$(MPa)	n_{1t}	n_{1t}/N_{1t}	n_{2t}	n_{2p}	n_{2p}	n_{2p}	n_{2p}
331.5–284.4 (H–L)	500	0.01	423,700	495,000	492,946	600,342	439,429
	12,500	0.25	250,400	375,000	332,926	454,805	236,220
	25,000	0.50	168,300	250,000	187,678	303,203	133,155
	37,500	0.75	64,500	125,000	70,446	151,602	58,170
284.4–331.5 (L–H)	125,000	0.25	37,900	125,000	70,446	151,602	47,459
	250,000	0.50	38,900	37,500	40,793	30,920	38,828
	375,000	0.75	43,400	25,000	30,622	20,613	23,798

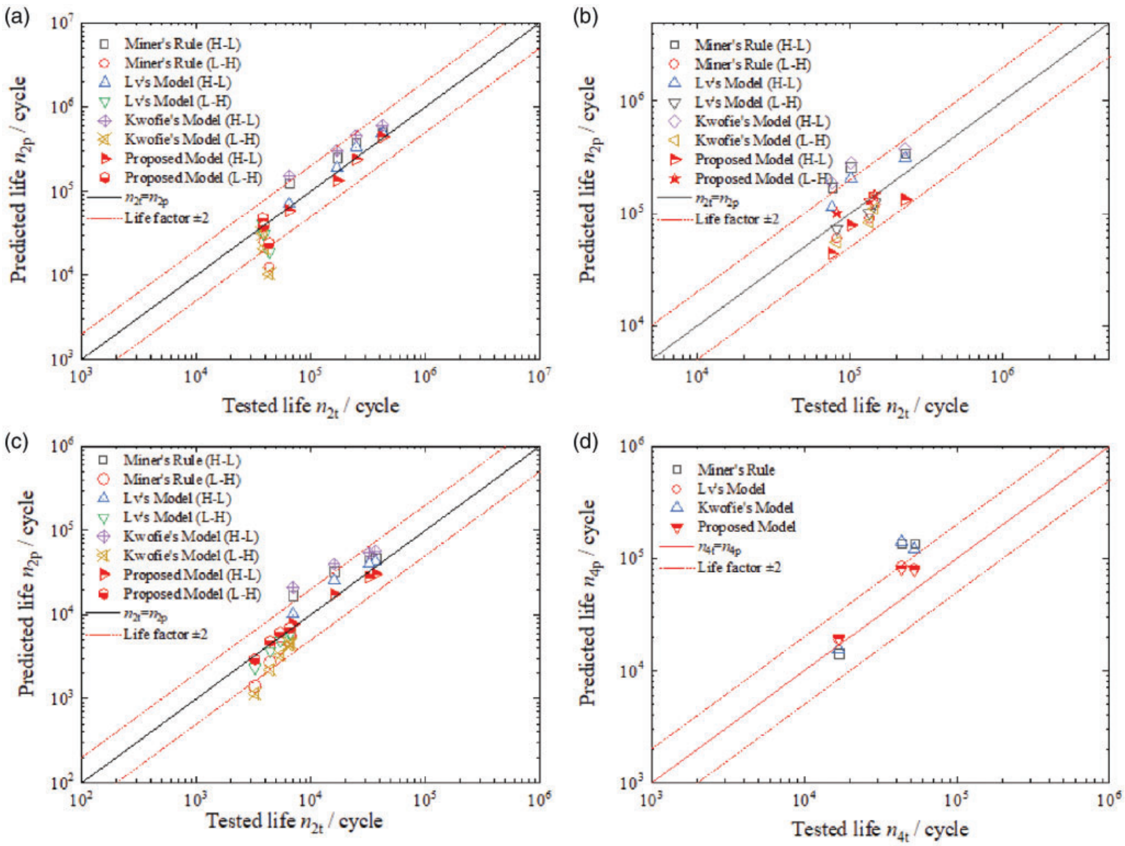

Figure 3. Predicted life vs. tested life of four materials. (a) C45, (b) Al 2024-T42, (c) 30CrMnSiA and (d) Al 6082-T6.

(2) Al 2024-T42

The experimental data of Al 2024-T42 was obtained under the high–low (200–150 MPa) and low–high (150–200 MPa) stress levels with $R = -1$, reported by Pavlou (2002). The fitted *S*–*N* curve of Al 2024-T42 based on the experimental data is shown in equation (30). The prediction results n_{2p} of the residual life cycles by the four models compared with experimental data n_{2t} are shown in Table 2 and Figure 3(b)

$$\ln \sigma_{\max} = -0.2732 \times \ln N + 8.554 \tag{30}$$

(3) 30CrMnSiA

The experimental data of 30CrMnSiA were obtained under two-level block loading fatigue tests, including (a) high–low (940–850 MPa) and low–high (850–940 MPa) stress levels with mean stress $\sigma_m = 450$MPa (the fitted *S*–*N* curve based on the corresponding data is given in equation (31)), and (b) high–low (836–732 MPa) and low–high (732–836 MPa) stress levels with mean stress $\sigma_m = 250$MPa (the fitted *S*–*N* curve based on the corresponding data is given in equation (32)),

Table 2. Residual lives of Al 2024-T42 steel under two-level block loading.

Loading sequence	Experimental data			Miner's rule	Lv's model	Kwofie's model	Proposed model
σ_{max}(MPa)	n_{1t}	n_{1t}/N_{1t}	n_{2t}	n_{2p}	n_{2p}	n_{2p}	n_{2p}
200–150 (H–L)	30,000	0.2	228,700	344,000	311,055	374,397	132,961
	60,000	0.4	101,050	258,000	204,895	280,798	78,224
	90,000	0.6	76,050	172,000	113,762	187,199	43,485
150–200 (L–H)	86,000	0.2	144,500	120,000	125,806	110,257	145,342
	172,000	0.4	133,500	90,000	100,280	82,693	129,041
	258,000	0.6	81,700	60,000	72,847	55,129	100,919

reported by Fang et al. (2006). The prediction results n_{2p} of the residual life cycles by the four models compared with experimental data n_{2t} are shown in Table 3 and Figure 3(c)

$$\ln \sigma_{\max} = -0.04291 \times \ln N + 7.194 \tag{31}$$

$$\ln \sigma_{\max} = -0.06588 \times \ln N + 7.316 \tag{32}$$

(4) Al 6082-T6

The experimental data for the Al 6082-T6 were obtained under four-level block loadings, reported by Aid et al. (2011). The fitted S–N curve of Al 6082-T6 based on the experimental data is given in equation (33). There are three sets of loading sequences: (a) 240–260–280–305 MPa, (b) 305–280–260–240 MPa, and (c) 280–305–260–240 MPa. The prediction results n_{4p} of the residual life cycles by the four models compared with experimental data n_{4t} are shown in Table 4 and Figure 3(d)

$$\ln \sigma_{\max} = -0.1024 \times \ln N + 6.8 \tag{33}$$

In Figure 3, for the prediction results of the residual life by the proposed model and Lv's model compared with the experimental data for the four materials under two-stress and four-stress levels, almost all of them are without ± 2 life factor scatter bands. The predicted lives of Miner's rule and Kwofie's model show larger scatters than those of Lv's model and the proposed model. Furthermore, a deviation P_{error} is employed to qualify the prediction errors of the four models, given as

$$P_{error} = \frac{n_p - n_t}{n_t} \tag{34}$$

where n_p and n_t denote the residual life of the predictions and the test, respectively.

The prediction errors by Miner's rule, Lv's model, Kwofie's model, and the proposed model for the four materials are shown in Figure 4. It can be seen that the proposed model can provide the most reasonable prediction results over the other three models, and Kwofie's model shows the largest deviation compared with the other models in estimating the residual fatigue life. The Lv's

Table 3. Residual lives of 30CrMnSiA steel under two-level block loading.

Loading sequence	Experimental data			Miner's rule	Lv's model	Kwofie's model	Proposed model
σ_{max}(MPa)	n_{1t}	n_{1t}/N_{1t}	n_{2t}	n_{2p}	n_{2p}	n_{2p}	n_{2p}
940–850 (H–L)	1000	0.292	23,996	24,947	21,009	32,097	15,300
	1000	0.292	17,830	24,947	21,009	32,097	15,300
	1700	0.497	15,046	17,724	12,592	22,803	9520
	1700	0.497	11,558	17,724	12,592	22,803	9520
	1700	0.497	10,219	17,724	12,592	22,803	9520
	2400	0.702	7294	10,500	5749	13,510	5045
	2800	0.819	1339	6378	2725	8206	2890
	2800	0.819	3347	6378	2725	8206	2890
	2800	0.819	4228	6378	2725	8206	2890
836–732 (H–L)	1200	0.167	36,911	46,444	42,841	56,836	30,378
	1800	0.208	32,450	44,158	39,833	54,038	27,732
	3000	0.417	16,002	32,505	25,608	39,778	17,363
	5000	0.694	6969	17,061	10,109	20,878	7727
850–940 (L–H)	5000	0.142	2812	2857	3007	2213	3280
	10,000	0.284	2488	2384	2666	1847	3102
	20,000	0.568	1942	1438	1904	1114	2349
	30,000	0.851	920	496	938	384	1027
732–836 (L–H)	13,000	0.233	6608	5593	6072	4552	7027
	15,000	0.268	6501	5338	5879	4344	6925
	25,000	0.448	5400	4025	4838	3276	6117
	35,000	0.628	4428	2713	3684	2208	4813
	45,000	0.807	3254	1407	2341	1145	2943

Table 4. Residual lives of Al 6082-T6 steel under four-level block loading.

Loading sequence	Experimental data				Miner's rule	Lv's model	Kwofie's model	Proposed model
	n_{1t}	n_{2t}	n_{3t}	n_{4t}	n_{4p}	n_{4p}	n_{4p}	n_{4p}
(a)	103,000	26,258	19,427	16,800	14,136	18,930	15,455	29,339
(b)	10,950	19,427	26,258	52,500	136,098	81,334	120,682	23,750
(c)	19,427	10,950	26,258	43,400	136,098	86,051	140,799	24,673

model can also provide acceptable results in predicting the residual life, but it is less accurate than the proposed model.

In total, the Lv' model and proposed model can provide reasonable prediction results under two-level and four-level block loadings, and the Miner's rule and Kwofie's model show much larger

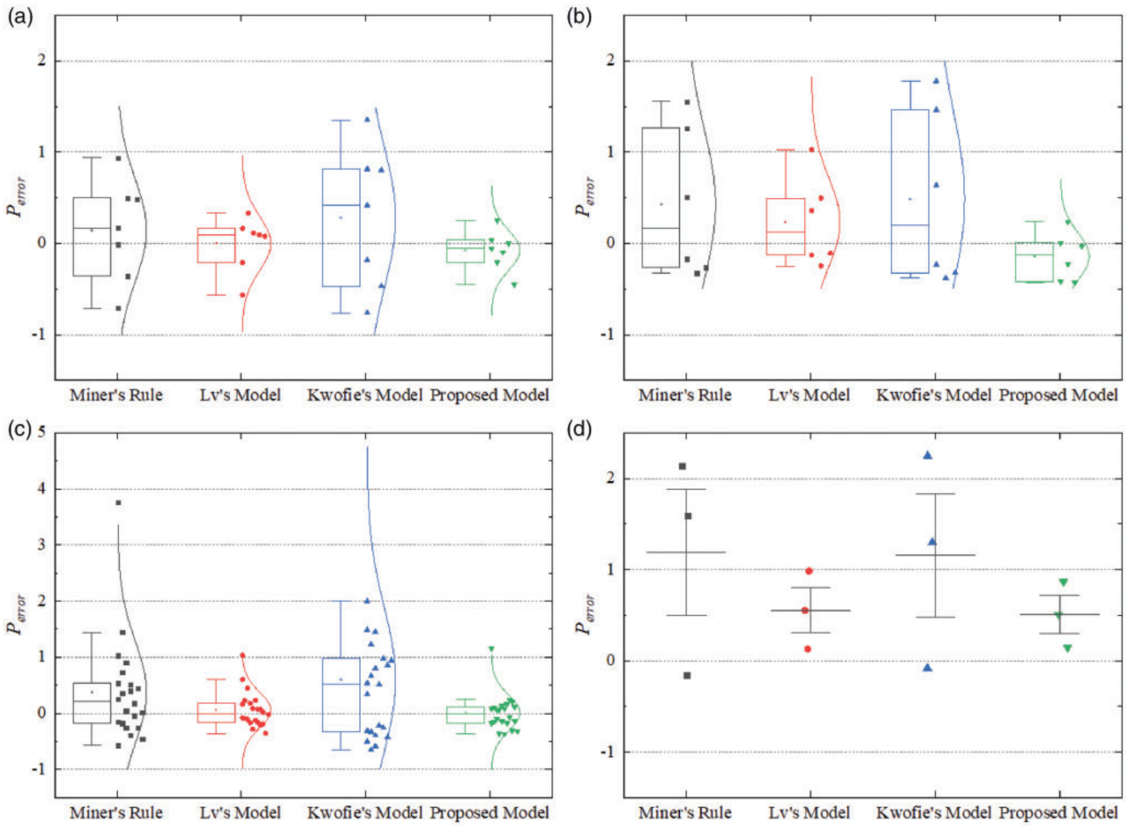

Figure 4. Prediction errors of four models. (a) C45, (b) Al 2024-T42, (c) 30CrMnSiA and (d) Al 6082-T6.

deviations compared with the experimental data. The proposed model shows the smallest deviations than the other three models under variable loadings, which considers the loading interaction and loading sequence based on the material memory, and it can satisfy the damage revolution in a rational way for different materials.

It should be noted that the proposed model will yield to Miner's rule under constant loading. If the stress is lower than the fatigue limit, which will be ignored by the proposed model. The proposed model is certified by the uniaxial experimental data, and it may be extended to the field of multiaxial fatigue criterion with the suitable equivalent parameters.

Conclusions

In this work, a non-linear cumulative damage model based on the degradation of material memory is presented. The accuracy and effectiveness of the proposed model are validated with four materials in estimating the residual life, and comparisons with Miner's rule, Lv's model, and Kwofie's model are also made. Some conclusions can be derived as follows:

(1) According to the comparisons with the predicted data of four models, the proposed model can provide the most reasonable predictions under cyclic loadings.

(2) When employing the proposed model, the relationship between the stress or equivalent factor and the fatigue life should be known. The proposed model originates from material memory, and it consists of a damage index and an interaction factor, which is sensitive to the loading levels and takes the loading history into account.

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