Contents lists available at ScienceDirect



International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci



# A nonlinear dissipative elastic metamaterial for broadband wave mitigation

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# ARTICLE INFO

Keywords: Nonlinear elastic metamaterial Meta-damping Wave attenuation Broadband

# ABSTRACT

Nonlinearity and dissipation are two important aspects in elastic metamaterials that hold the potential to provide a novel method for the control of elastic waves. Here, we present a nonlinear dissipative elastic metamaterial in a triatomic mass-spring chain to explore the interplay between nonlinearity and dissipation for broadband wave attenuation. In the study, nonlinear stiffness is considered in the spring between the primary and secondary triatomic masses, and a damper is implemented in parallel to the tertiary spring. Various numerical tests on transient wave propagations are conducted, and show that the proposed design with properly selected nonlinear and damping parameters can generate broader wave attenuation regions compared with corresponding linear dissipative metamaterials and nonlinear non-dissipative metamaterials. We quantify the effects of nonlinearity and material damping in higher harmonic wave generations and wave energy absorption using narrow band incidences and demonstrate the application of the nonlinear dissipative triatomic lattice for blast wave mitigations. This work provides a novel approach to design materials capable of suppressing blast-induced shock waves or impact generated pulses that can cause severe local damage to nearby structures.

# 1. Introduction

Metamaterials are engineered materials that possess unique dynamic effective properties due to their specially designed microstructures rather than the naturally occurring materials used in their construction. [1-5]. In recent years, electromagnetic metamaterials have been developed for microwave absorbers, optical energy harvesting and sensor applications [6-11], meanwhile a great deal of theoretical, numerical and experimental research has been conducted on elastic metamaterials (EMs) with novel applications such as elastic wave absorbers, negative refractive indices, cloaking and superlensing [5]. The primary components in the design of metamaterials are locally resonant (LR) inclusions at subwavelength scales such that the EM can be regarded as an effective continuum media exhibiting, for example, negative effective mass density and/or negative effective moduli [12-17]. LR allows for the manipulation or suppression of low-frequency waves without requiring large additional masses or unrealistic structural scales [18-20]. While there are clear advantages of the LR mechanism, there remain a number of limitations that have yet to be overcome, i.e. band gaps produced by LR are generally narrow and fixed over specific frequencies.

Up until now, the primary research focus has been aimed at the improvement or optimization of linear EMs [21,22] which are typically based on a LR mechanism [23–27] to achieve broadband structural vi-

bration suppression and seismic wave protection with minimal weight addition [28-35]. The common approaches that have been suggested are based on either passive or active design stratergies. For example, active EMs with tunable or adaptive resonators built with shunted piezoelectric materials have been proposed for broadband wave attenuation [36]. In those material systems, complex electrical control modules are usually involved. One the other hand, passive approaches have been extensively applied by taking the advantages of simplicity and robustness. To enlarge wave/vibration attenuation frequency regions, multiple or gradient resonator integrations have been suggested [22,37]. Additionally, including materials with strongly dissipative properties into EMs with single or multiple resonators is one of the favourite strategies in the EM design [33,37]. Broadband elastic wave suppression has been demonstrated numerically and experimentally with this approach, thanks to the interaction between the resonant motion of the local resonators and dissipation [21,22].

Besides the linear regime, elastic waves propagating in metamaterials with nonlinear characteristics has seen an increasing amount of research attention and shown to possess a number of interesting and unique properties such as wave coupling, subharmonic generation, nonlinear dioding, discrete breathing, solitary wave propagation and topological insulators [38–42]. Previous work on nonlinear elastic metamaterials (NEMs) has been primarily based on discrete chains and granular

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https://doi.org/10.1016/j.ijmecsci.2019.105159

Received 17 July 2019; Received in revised form 10 September 2019; Accepted 12 September 2019 Available online 13 September 2019

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crystals due to the Hertzian contact phenomenon [43,44]. Strong nonlinearity has been shown to increase the velocity of elastic waves and to modify band gap properties in nonlinear pieriodic medium [45–48]. For example, to broaden the wave attenuation frequency region, an ultralow and ultra-broadband nonlinear acoustic metamaterial has been proposed to combine conventional band gaps with chaotic bands [47,48]. In addition, two one-dimensional (1D) nonlinear mass-spring chains featuring diatomic and tetratomic microstructures have been investigated, which demonstrated the expansion of the bandwidth for elastic wave suppression. However, NEMs with multiple nonlinear dissipative resonators have yet to be studied, and the design to achieve broadband and strong wave attenuations with proper selections of nonlinear and damping coefficients has not been reported.

In the following work, we propose a nonlinear dissipative triatomic lattice metamaterial capable of broadband elastic wave attenuation. In the design, a nonlinear spring is equipped between the primary and secondary triatomic masses, and a damper is implemented in parallel to the tertiary spring. We first develop an analytical model to characterize dispersion properties of the nonlinear metamaterial based on linearization assumptions. The band gaps produced by the nonlinear nondissipative triatomic lattice are qualitatively estimated, and the nonlinear hardening effect is clearly seen. Several numerical simulations on transient wave propagations through the nonlinear dissipative triatomic lattice are then conducted with both narrow and broadband incidences. The results show that by properly selecting nonlinear and damping parameters, the proposed design can generate broader wave mitigation regions compared with corresponding linear dissipative metamaterials and non-dissipative nonlinear metamaterials. The effects of nonlinearity and material damping in higher harmonic wave generations are investigated in further detail. Finally, the application of the nonlinear dissipative triatomic lattice for blast wave mitigation is demonstrate numerically. This work provides a design guideline for building nonlinear dissipative materials to suppress blast-induced shock waves or impact generated pulses which are broadband in nature.

# 2. Nonlinear triatomic mass-spring lattice and its dispersion relations

Fig. 1 shows the schematic of the nonlinear triatomic mass-spring lattice proposed in this study. As shown in the microstructure illustrated in the rectangular region enclosed by red dashed lines, each of its unit cell contains two local resonators and is coupled to each of its adjacent unit cells by a linear spring with stiffness  $k_0$ . The three ideallyrigid masses that make up each of the unit cells are denoted as  $m_0, m_1$ , and  $m_2$ . The nonlinear Duffing oscillator is directly connected to the host lattice, which consists of a mass,  $m_1$  and a nonlinear spring with cubic nonlinear stiffness,  $k_1 \Delta + k_{nl} \Delta^3$  ( $\Delta$  represents the displacement difference at the two ends of the spring). The other mass-spring oscillator connected to the Duffing oscillator in the unit cell consists of a mass  $(m_2)$ , a linear spring with stiffness  $k_2$ , and a dashpot with damping coefficient  $c_2$ . The displacements of  $m_0, m_1$ , and  $m_2$  in the *n*-th unit cell are denoted by  $u_n, v_n$ , and  $w_n$ , respectively. The equations of motion for the *n*-th cell can be expressed as

$$\begin{split} m_{0}\ddot{u}_{n} &= k_{0}(u_{n+1} + u_{n-1} - 2u_{n}) + k_{1}(v_{n} - u_{n}) + k_{nl}(v_{n} - u_{n})^{3}, \\ m_{1}\ddot{v}_{n} &= k_{1}(u_{n} - v_{n}) + k_{nl}(u_{n} - v_{n})^{3} + k_{2}(w_{n} - v_{n}) + c_{2}(\dot{w}_{n} - \dot{v}_{n}), \end{split}$$
(1)  
$$\begin{split} m_{2}\ddot{w}_{n} &= k_{2}(v_{n} - w_{n}) + c_{2}(\dot{v}_{n} - \dot{w}_{n}). \end{split}$$



**Fig. 2.** Dispersion relations of the nonlinear triatomic mass-spring lattice with different normalized nonlinear coefficients.

To calculate the dispersion relations of the nonlinear triatomic massspring lattice, we employ a first-order time harmonic approximation such that the nonlinear spring is approximately described by an equivalent linear spring with

$$k_{\rm eq} = k_1 + 3k_{nl}(\Delta)^2.$$
 (2)

Note that Eq. (2) is derived from the system at its static equilibrium. When applying it to dynamics where higher-order time harmonics are ignored,  $\Delta$  represents the effective amplitude of the displacement difference at the two ends of the nonlinear spring. Although the approach cannot accurately capture the wave behavior of the nonlinear triatomic mass-spring lattice, it will provide qualitative estimations on wave band gap frequencies due to stiffening characteristics of nonlinear springs [46].

Given the linear nature of this simplified lattice model, applying the Bloch theorem to the periodic lattice requires that

$$\mathbf{u}_{n+1} = \mathbf{u}_n e^{i\kappa a},\tag{3}$$

where  $\kappa$  and *a* represent the wavenumber and lattice constant, respectively, and  $\mathbf{u}_n = [u_n \quad v_n \quad w_n ]^T$ . Combining Eq.'s (1–3), the dispersion relation can be obtained as

$$\mathbf{K}' - i\omega\mathbf{C} + \omega^2\mathbf{M} = 0, \tag{4}$$

where

$$\begin{split} \mathbf{M} &= \begin{bmatrix} m_0 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_2 \end{bmatrix}, \\ \mathbf{K} &= \begin{bmatrix} k_0 \left(2 - e^{i\kappa a} - e^{-i\kappa a}\right) + k_{eq} & -k_{eq} & 0 \\ -k_{eq} & k_2 + k_{eq} & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix}, \\ \mathbf{C} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & c_2 & -c_2 \\ 0 & -c_2 & c_2 \end{bmatrix}. \end{split}$$

Fig. 2 shows the dispersion relations of the nonlinear triatomic massspring lattice with different degrees of nonlinearities. It can be observed that, according to Eq. (4), in the presence of material damping ( $c_2 \neq 0$ ), the frequency becomes complex given real wavenumbers.

> Fig. 1. Schematic of the nonlinear triatomic massspring lattice.



In our dispersion investigations, we focus on the effects of nonlinearities on band gap frequencies. Therefore, the damping coefficient,  $c_2$ , is imposed to zero in Fig. 2. In the calculations, the masses  $m_0$ , and spring constants  $k_0$  are selected as 0.002 kg and 7.9 × 10<sup>6</sup> N/m, respectively. Other mass and linear spring constant ratios are selected as  $m_1/m_0 = 5$ ,  $m_2/m_0 = 20$ ,  $k_1/k_0 = 0.2$  and  $k_2/k_0 = 0.05$  for optimized wave attenuation performance [37]. We define a normalized nonlinear coefficient,  $\sigma = \tilde{\Delta}^2 k_{nl}/k_1$ , with  $\tilde{\Delta}$  being the input displacement amplitude and  $k_{nl}a^2/k_0 = 15$ . In Fig. 2, we plot the normalized frequency,  $\Omega = \omega / \sqrt{k_0 / m_0}$  with different normalized wavenumbers,  $\kappa \pi / a$ . It can be found from the figure that two gaps in frequency are generated by the local resonances of the two resonators. By increasing the normalized nonlinear coefficient,  $\sigma$ , the band gap occupying lower frequencies is unchanged, whereas the second band gap is shifted to higher frequencies with much broader bandwidths. This behavior is understandable, as the band gap occupied at lower frequencies is mainly caused by the resonance of the linear resonator,  $m_2 - k_2$  [37]. Adding nonlinearity only modifies the coupling between  $m_1 - k_1$ . Therefore, the band gap occupied at lower frequencies will not be effectively affected by nonlinearity.

On the other hand, the nonlinear resonator is mainly responsible for the other band gap. Increasing the nonlinearity produces an equivalent spring with higher stiffness (Eq. (2)), resulting in broader band gaps occupying higher frequencies. Although the total wave attenuation frequency region is enhanced with the increase of nonlinearity, a pass band still exists between the two band gaps and becomes broader with larger nonlinearities, where the propagation of elastic waves is uninhibited. This exhibits a major drawback of the nonlinear non-dissipative triatomic mass-spring lattice for broadband shock wave attenuation. To address this disadvantage, we will investigate the interplay between nonlinearity and dissipation by conducting time-dependent analyses subsequently.

# 3. Transient analysis of the nonlinear dissipative triatomic mass-spring lattice

To explore the wave attenuation ability of the triatomic nonlinear dissipative metamaterial, time domain analyses are conducted on a 1D lattice system, where the metamaterial with 15 unit cells is inserted into the middle portion of a linear single atomic mass-spring chain characterized by  $m_0$  and  $k_0$  (Fig. 3). A displacement boundary condition is applied to the first unit cell of the lattice system to generate a longitudinal wave. In the study, the fourth-order Runge–Kutta method is adopted based on MATLAB for calculating the transient displacement fields within the lattice. Reflected waves from boundaries of the single atomic mass-spring chain that contains 1000 unit cells can be successfully separated from the transmitted waves in the time-domain signals for the frequencies of interest [48].

### 3.1. Wave transmission spectra

The transmission spectra of the nonlinear dissipative triatomic lattice is obtained numerically by extracting the displacement from the single atomic chain (550th unit cell) in the transmitted domain of the lattice system. Swept-sine signals are applied to the first unit cell to generate incident longitudinal waves. Fig. 4 shows the transmitted signals in the frequency domain normalized to the incident signal. In the calculations, mass and linear spring constants are kept the same as those used in Fig. 2, and different nonlinear and damping coefficients are employed for comparisons.



**Fig. 4.** Wave transmission spectra of the nonlinear dissipative triatomic massspring lattice with different normalized nonlinear and damping coefficients: (a)  $\tau_2 = 0.0072$ ; (b)  $\tau_2 = 0.072$ ; (c)  $\tau_2 = 0.72$ .

In the presence of an extremely small normalized damping coefficient ( $\tau_2 = c_2/\sqrt{k_2 m_2}$ =0.0072, Fig. 4a), wave attenuation is clearly seen in two separate frequency regions. When the normalized nonlinear coefficient,  $\sigma$ , is increased from 0 to 1, the wave attenuation region occupied at lower frequencies is kept unchanged, whereas the other highfrequency wave attenuation region is shifted to higher frequencies with much broader bandwidths. This behavior agrees reasonably well with the prediction in Fig. 2 for different nonlinearities. As also expected from Fig. 2, a pass band between the two wave attenuation regions still remains, no matter what kinds of different nonlinear coefficients,  $\sigma$ , are used. However, this pass band can be suppressed by adding moderate material damping properties to the nonlinear triatomic mass-spring lattice. Fig. 4b shows the transmitted signals from numerical simulations, when  $\tau_2 = 0.072$ . As shown in the figure, the wave attenuation in this pass band frequency region becomes much stronger compared with those in Fig. 4a. Note that there still exists a small amount of energy leaking from the nonlinear section of the lattice, which can be sufficiently suppressed by adding more unit cells. Instead of doing this, a dashpot with extremely large damping coefficient,  $\tau_2 = 0.72$ , is implemented as shown in Fig. 4c. It can be seen from the figure that the original pass band completely disappears, resulting in a single strong wave attenuation region. When  $\sigma = 1$ , the wave attenuation region is very broadband occupying normalized frequencies from  $\Omega = 0.05$  to 0.6. This broadband wave attenuation frequency region highlights the interplay between nonlinearity and material damping: nonlinearity contributes to the migration of the original high-frequency wave attenuation region to much higher and broadband frequencies and the material damping mitigates the pass band between the two original wave attenuation frequency regions. The effects of nonlinearity and material damping parameter in the nonlinear dissipative triatomic lattice on wave attenuation frequency regions have also been summarized in Table 1.

In order to quantitatively discuss the physical phenomena of nonlinearity, the wave transmission spectra with continuously varied



Fig. 3. Schematic of the nonlinear dissipative triatomic mass-spring lattice in transient analysis.

#### Table 1

Wave attenuation frequency regions with different nonlinearities and material damping parameters in the nonlinear dissipative triatomic lattice.

$\tau_2$	σ 0.00	0.04	0.25	1.00
0.0072	(0.05,0.13)	(0.05,0.13)	(0.05,0.13)	(0.05,0.13)
	(0.25,0.50)	(0.25,0.51)	(0.26,0.55)	(0.30,0.63)
0.072	(0.05,0.13)	(0.05,0.13)	(0.05,0.13)	(0.05,0.13)
	(0.25,0.50)	(0.25,0.51)	(0.25,0.55)	(0.25,0.63)
0.72	(0.05,0.50)	(0.05,0.51)	(0.05,0.55)	(0.05,0.63)



**Fig. 5.** Wave transmission spectra by continuously varying the normalized nonlinear coefficient,  $\sigma$ , of the nonlinear dissipative triatomic lattice with different material damping coefficients.

normalized nonlinear coefficient,  $\sigma$  has been shown in Fig. 5. In particular, the normalized frequency  $\Omega$  is selected as 0.25 and 0.60, respectively, which are located within the lower and upper boundaries of the second bandgap of the linear nondissipative triatomic lattice, to show changes of wave attenuation regions. It can be concluded from the figure that (a) in the presence of the small damping coefficient,  $\tau_2 = 0.0072$ , when the normalized nonlinear coefficient,  $\sigma$ , gradually increases from 0.05 to 1, the normalized amplitudes of transmitted displacement signals at  $\Omega\Omega = 0.25$  gradually increase from 0.1 to 0.9, indicating that the lower boundary of the second band gap shifts to higher frequencies and the original stop band becomes the pass band at  $\Omega\Omega = 0.25$ .



Whereas, the normalized amplitudes of transmitted displacement signals at  $\Omega\Omega = 0.60$  gradually decrease from around 0.8 to 0.2, implying that the upper boundary of the second band gap also shifts to higher frequencies and the original pass band becomes the stop band at  $\Omega\Omega = 0.60$ . (b) in the presence of the large damping coefficient,  $\tau_2 = 0.72$ , when the normalized nonlinear coefficient,  $\sigma$ , gradually increases from 0.05 to 1, the normalized amplitudes of transmitted displacement signals at  $\Omega\Omega = 0.25$  remains around zero, which means that the wave attenuation performance becomes unaffected near the lower boundary of the second band gap. At the same time, the normalized amplitudes of transmitted displacement signals at  $\Omega\Omega = 0.60$  are almost same with that when the small damping coefficient,  $\tau_2 = 0.0072$ , is presented. Therefore, waves with frequency components occupying the entire frequency region,  $\Omega\Omega = 0.05$ –0.63, can be blocked by the nonlinear dissipative triatomic lattice. In the study, the ratio between the propagated wavelength and lattice constant is from 65 to 10 for this frequency range. Consequently, the nonlinear dissipative triatomic lattice can be used to represent as an elastic metamaterial.

# 3.2. Narrow-band incidence

To further show details of interactions between nonlinearity and material damping of the nonlinear dissipative triatomic lattice, timedependent analyses with narrow-band incidences are conducted. The non-dimensionalized frequency region of particular interest,  $\Omega = 0.15 \sim 0.35$ , is where the pass bands of both the linear and nonlinear triatomic lattices can be suppressed by increasing the material damping. Without loss of generality, incident signals are first selected as sine waves with the frequency  $\Omega = 0.3$ . The parameters of the system studied herein are the same as those applied in Fig. 4 (the schematic illustration is shown in Fig. 3).

Fig. 5 shows phase diagrams of the outermost mass,  $m_0$ , in the first unit cell of the triatomic lattice using response trajectories between the displacement and velocity. In Figs. 6a–c, the normalized nonlinear coefficient is kept the same as  $\sigma = 1$ , and the damping coefficient,  $\tau_2$ , is selected as 0.0072, 0.072 and 0.72, respectively. For comparison, phase diagrams of the outermost mass in the first unit cell of the linear dissipative triatomic lattice are illustrated in Figs. 6d–f, where the damping coefficient,  $\tau_2$ , is again selected as 0.0072, 0.072 and 0.72, respectively. For the linear triatomic lattice (Figs. 6d–f), displacement and velocity responses are almost the same when  $\tau_2$  is increased from 0.0072 to 0.72, showing stable single-frequency responses. Whereas, in the presence of nonlinearity, multiple nested rings containing high-frequency responses are clearly seen in Fig. 6a, when the triatomic lattice is prescribed a small damping coefficient. By increasing the damping coefficient,  $\tau_2$  to 0.072, the frequency up-conversion can be weakened (Fig. 6b). When

> **Fig. 6.** Normalized phase diagrams of the outermost mass in the first unit cell of the linear and nonlinear triatomic lattices represented by the response trajectories between the displacement and velocity. (a–c): Nonlinear dissipative triatomic mass-spring lattice: (a)  $\tau_2 = 0.0072$ ; (b)  $\tau_2 = 0.072$ ; (c)  $\tau_2 = 0.72$ . (d–f): Linear dissipative triatomic mass-spring lattice: (d)  $\tau_2 = 0.0072$ ; (e)  $\tau_2 = 0.072$ ; (f)  $\tau_2 = 0.72$ .



Fig. 7. Waterfall plots of power spectra of the first unit cell in the nonlinear dissipative triatomic lattice with different normalized nonlinear coefficients: (a)  $\tau_2 = 0.072$ ; (b)  $\tau_2 = 0.072$ ; (c)  $\tau_2 = 0.72$ .

the lattice is prescribed an extremely large material damping coefficient,  $\tau_2 = 0.72$ , the frequency conversion effects due to nonlinearity are almost entirely lost (Fig. 6c), and the displacement and velocity responses are close to those of the linear dissipative triatomic lattice.

To more clearly see the frequency conversions, Fig. 7 shows waterfall plots of the power spectra of the first unit cell in the nonlinear dissipative triatomic lattice by continuously changing the normalized nonlinear coefficients. In Figs. 7a–c, the damping coefficient,  $\tau_2$ , is selected as 0.0072, 0.072 and 0.72, respectively, and 10-peak tone-burst signals with the central frequency  $\Omega = 0.3$  are used. If small damping coefficients are applied ( $\tau_2 = 0.0072$  in Fig. 7a), the frequency conversion to the triple of the input frequency becomes strong when  $\sigma > 0.2$ . Fur-

thermore, we start to see the chaotic behavior, when  $\sigma$  approaches 0.4. Increasing the damping coefficient will compromise the nonlinear behavior and weaken frequency up-conversions. For example, in Fig. 7b, where  $\tau_2 = 0.072$ , the frequency conversion begins to arise when  $\sigma > 0.4$ . The chaotic behavior is still observed but becomes extremely weak, when  $\sigma$  approaches 0.7. In addition, if extremely large damping coefficients are applied ( $\tau_2 = 0.72$  in Fig. 6c), both the frequency conversion and chaotic behavior can be nearly switched off. Therefore, carefully prescribed degrees of nonlinearity and material damping coefficients could be a potential solution in controlling amplitudes of higher harmonic waves generated in the nonlinear triatomic lattice.



**Fig. 8.** Frequency spectra of wave energy intensities flowing through all the outermost masses in linear and nonlinear triatomic lattices: (a, c, e): Nonlinear dissipative triatomic mass-spring lattice: (a)  $\tau_2 = 0.0072$ ; (c)  $\tau_2 = 0.072$ ; (e)  $\tau_2 = 0.72$ . (b, d, f): Linear dissipative triatomic massspring lattice: (b)  $\tau_2 = 0.0072$ ; (d)  $\tau_2 = 0.072$ ; (f)  $\tau_2 = 0.72$ .



**Fig. 9.** Time domain displacement signals extracted from the background linear single atomic lattices. (a and b) Linear dissipative triatomic mass-spring lattice: (a)  $\tau_2 = 0.0072$ ; (b)  $\tau_2 = 0.072$ ; (c and d) Nonlinear dissipative triatomic mass-spring lattice: (c)  $\tau_2 = 0.0072$ ; (d)  $\tau_2 = 0.072$ .

It should be noted that Figs. 5 and 6 only show the dynamic responses of the first unit cell. In Fig. 8, frequency spectra of wave energy intensities flowing through all the outermost masses in the nonlinear triatomic lattice are illustrated. In the studies, 10-peak tone-burst signals with the central frequency  $\Omega = 0.3$  are used again. For comparison, frequency spectra of the linear triatomic lattice are also shown in Fig. 8a, c and e. The normalized nonlinear coefficients, ( $\sigma = 1$ ), are kept constant among all the cases in Fig. 8b, d and f. In Fig. 8a and b, the damping coefficient,  $\tau_2$ , is selected as 0.0072. For the linear triatomic lattice (Fig. 8a), the incident wave is quickly decayed without frequency conversions after entering the lattice. This is expected, as the incident frequency is inside band gap frequencies (see Figs. 2 and 4). On the other hand, for the nonlinear triatomic lattice (Fig. 8b), the incident wave is slowly decayed and partially converted to waves at triple of the incident wave frequencies after entering the lattice. The converted high-frequency waves propagate uninhibited along the nonlinear lattice, as their frequencies are outside nonlinear band gap frequencies. As a result, the transmitted wave contains frequency components of both the fundamental and triple frequencies. This behavior agrees reasonably well compared with Fig. 4a. In Fig. 8c and d, the damping coefficient,  $\tau_2$ , is increased to 0.072. In Fig. 8c, the wave attenuation behavior along the linear triatomic lattice at the incident frequency is almost the same with that in Fig. 8a, where  $\tau_2 = 0.0072$ . Whereas for the nonlinear triatomic lattice (Fig. 8d), the incident wave experiences a much stronger decay compared with that in Fig. 8b and converted high-frequency waves have much smaller amplitudes, due to the presence of the increased damping coefficient. Again,

the converted high-frequency waves from several unit cells facing the incident wave will totally transmit through the nonlinear triatomic lattice. In Fig. 8e and f, the damping coefficient,  $\tau_2$ , is increased to 0.72. Similarly, the wave attenuation along the linear triatomic lattice at the incident frequency has no change compared with those in Fig. 8a and c. Meanwhile, the nonlinear triatomic lattice shows excellent wave attenuation where the incident wave can only travel several unit cells of the nonlinear triatomic lattice and then disappears almost completely. It is interesting to note that the waves at fundamental frequencies have not been converted to high-frequency waves due to the presence of the extremely large material damping coefficient. In addition, it may be noticed that the wave attenuation performance of the nonlinear triatomic lattice at incident frequencies already reaches the strength of the locally resonant linear triatomic lattice.

It is necessary to mention that the approaches used to obtain the nonlinear dynamic characteristics of the first unit cell demonstrated in Figs. 6 and 7 are applicable to other unit cells in the nonlinear triatomic lattice. Special attention should be given however that when waves pass through the lattice, their amplitudes will vary from one unit cell to the other unit cells, as illustrated in Fig. 8. Therefore, the phase diagrams and waterfall plots of power spectra of other unit cells in the nonlinear dissipative triatomic lattice would possess similar trends as those shown in Figs. 6 and 7, but with different nonlinear and damping coefficients at transition points of the frequency conversions. It should also be noted that the nonlinear hardening effects of the nonlinear dissipative triatomic lattice are still preserved such that the broadband highfrequency wave attenuation is observable (Fig. 4c). However, attenuation at high frequencies inside the nonlinear band gap ( $\Omega = 0.45$ –0.7) becomes weaker as the wave amplitudes gradually decrease while passing through the lattice. To overcome this drawback, gradient nonlinear dissipative triatomic lattices could be a simple solution for better wave attenuation performances at broadband frequencies.

Besides the low transmittances, the nonlinear triatomic lattices also support superior wave absorptions when compared with the linear counterparts. To demonstrate this, transmitted and reflected wave signals are extracted from the linear single atomic background lattices (550th unit cell for transmissions and 300th unit cell for reflections). Linear or nonlinear triatomic lattices are embedded into the middle portion of the linear single atomic lattices. The incident signals and material parameters are left unchanged as those in Fig. 8. Fig. 8a and b show transmitted and reflected wave signals, when linear and nonlinear triatomic lattices with extremely small damping coefficients ( $\tau_2 = 0.0072$ ) are involved, respectively. Comparing the two figures, it is clear that most of the energy is reflected back due to the presence of the linear triatomic lattice. By including the nonlinear triatomic lattice, transmitted waves become dispersive and possess much larger amplitudes. As expected, frequency conversions are clearly observed in both transmitted and reflected signals. It should also be noted that the energy dissipation in both of the two cases is negligibly small. By increasing  $\tau_2$  to 0.072, Fig. 8c and d show transmitted and reflected waves signals with linear and nonlinear dissipative triatomic lattices, respectively. As shown in Fig. 9c, the transmitted signal is almost the same as that in Fig. 9a, whereas the amplitude of reflected signal is slightly decreased, indicating that a small portion of the incident energy is absorbed by the linear dissipative triatomic lattice. By properly combining nonlinearity and the material damping (Fig. 9d), the transmitted wave can be almost entirely suppressed with close to zero amplitudes. In addition, the amplitude of the reflected signal is decreased again, implying that more incident energy is absorbed by the nonlinear dissipative triatomic lattice compared with the linear dissipative triatomic lattice.

## 3.3. Blast wave incidence

In this section, we employ a blast wave signal to evaluate impact wave mitigation abilities of the nonlinear dissipative triatomic lattice. Time-dependent simulations are conducted based on a nonlinear dissipative triatomic lattice containing 40 unit cells. A displacement boundary condition representing an incident blast signal is applied to the first unit cell at the left-most boundary of the lattice, where the applied displacement is given by  $X = X_{\max}e^{\frac{t-t_0}{t_d}}$ , with  $t_0 = 0.5$  ms and  $t_d = 0.1$  ms. The last unit cell at the right boundary of the lattice is left free.

Fig. 10 shows snapshots of the displacement responses of the 40 unit cells in the nonlinear dissipative triatomic lattice at different times. In



**Fig. 10.** Displacement responses of the 40 unit cells in the dissipative triatomic lattices with different normalized nonlinear coefficients. (a–d): (a)  $\sigma$  = 0; (b)  $\sigma$  = 0.04; (c)  $\sigma$  = 0.25; (d)  $\sigma$  = 1.0.

the figures, the vertical axis and color represents the normalized displacements. In the simulations, mass, spring and damping parameters are the same as those used in Fig. 8b, and the normalized nonlinear coefficient,  $\sigma$ , is selected as 0, 0.04, 0.25, and 1 in Figs. 10a–d, respectively. As illustrated in Fig. 10a, when the blast wave propagates along the linear dissipative triatomic lattice, the sharp peak in the wave front becomes dispersive and gradually decreases along the lattice. The blast wave is finally reflected, after reaching the free boundary. By increasing the normalized nonlinear coefficient,  $\sigma$ , to 0.04 (Fig. 10b), the sharp peak in the wave-front decays faster along the lattice and the amplitude becomes smaller at the free boundary compared with that of the linear dissipative triatomic lattice. By further increasing the normalized nonlinear coefficient,  $\sigma$ , to 0.25 (Fig. 10c), the sharp peak is quickly decayed in a few unit cells and the blast wave transmitted to the free end is nearly negligible. Furthermore, it should be noticed that the blast wave is concentrated in the first unit cells and takes more time to fully decayed. Again, if  $\sigma = 1.0$  (Fig. 10d), the blast wave is almost completely prohibited from propagating along the lattice, which will result in near zero transmittances. However, it will take much more time to completely dissipate the concentrated blast wave energy in the first unit cells.

#### 4. Conclusions

In summary, we propose and numerically evaluate a nonlinear dissipative triatomic lattice metamaterial, by equipping nonlinear springs between the primary and secondary triatomic masses and implementing dampers in parallel to tertiary springs, with the aim to mitigate broadband elastic waves. Wave band gaps of nonlinear non-dissipative triatomic lattices are first approximated by linearized dispersion analyses. We characterize the dynamic time-dependent responses of the nonlinear dissipative triatomic lattice using the fifth-order Runge-Kutta method. The calculated transmission spectra show that the proposed design with properly selected nonlinear and damping parameters can generate broader wave attenuation frequency regions when compared with corresponding linear dissipative and nonlinear non-dissipative counterparts. Furthermore, higher harmonic waves generated by the nonlinear dissipative triatomic lattice are quantified with different nonlinear and damping parameters using narrow band excitations. In addition, we find that the nonlinear dissipative triatomic lattice can provide stronger wave absorption than the corresponding linear dissipative triatomic lattice at the excitation frequency used in the study. Finally, the nonlinear dissipative triatomic lattice is numerically tested for blast wave mitigations which present a persistent challenge as they occupy a broad range of frequencies by nature. The suggested design and its parameter studies should shed light on the development of nonlinear mechanical metamaterials for elastic wave manipulation in general, impact and blast wave mitigation in particular.

# Acknowledgments

This work is supported by the Air Force Office of Scientific Research under Grant No. FA9550-18-1-0096 with Program Manager Dr. Byung-Lip (Les) Lee.

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